Assessing the impact of on-net and cross-net bill-neutral price changes: example of the French mobile market

Nicolas Curien
Conservatoire national des arts et métiers
CNAM, 292, rue Saint Martin, 75003 Paris, France
nicolas.curien@academie-technologies.fr

Natalia Shutova
Senior Consultant, TERA Consultants
39, rue d'Aboukir, 75002 Paris, France
shutova@tera.fr

Paris Center for Law and Economics
CRED, 26 rue des Fossés Saint-Jacques, 75005 Paris, France

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Abstract

This article proposes a theoretical model of a mobile telephony market with the purpose to study the impact of on-net /off-net (intra-operator/ inter-operator) price differentiation. The proposed model specification allows us not only to find social optimum and equilibrium but also to compare the impact of two types of price cuts on operators and consumers: all-net price cuts, equivalent to raising volume ceilings in packages, and on-net price cuts, equivalent to introducing unlimited on-net calls. It appears that all-net price cuts are more beneficial to consumers. The model is then numerically calibrated based on the data of the French mobile market in 2003; it is used to get quantitative results on the impact of price changes. We find that on the French market, where in 2004 two biggest operators opted for on-net price cuts and the third one – for all-net cuts, all-net price cuts by all operators would have been preferable: the increase in the welfare would have been 70% higher.

1 Introduction

A number of theoretical works have been devoted to the issue of competition between electronic communications networks. Among this rich literature, we shall only review here the papers that cope with the issue of price differentiation between traffic directions, typically the *on-net/off-net* (intra-network/inter-network) differentiation. Most of the proposed models study duopolistic markets with either symmetric or asymmetric operators. Only several works deal with competition between more than two operators, either in the general case as in Hoernig (2009), or in particular cases: the one of a symmetric oligopoly as in Calzada and Valetti (2008) and the one of an asymmetric oligopoly of four operators – including two big and two small operators – in Dewenter and Haucap (2005). Despite the variety of models, there is a general agreement that the ability to differentiate price with respect to traffic direction significantly alters the characteristics of competitive equilibrium.

The key findings are as follows.

- On a symmetric duopolistic market:
  - in the short term, the ability to differentiate prices according to call direction intensifies competition and hence drives down the average price level and increases the consumers' surplus (cf. Laffont et al, 1998-a);
  - in the long term, an on-net/off-net price differentiation may reduce the degree of connectivity between the two networks without modifying their respective market shares, leading to a drop in the consumers' surplus (cf. Jeon et al, 2004).

- On an asymmetric market:
  - an entrant whose coverage is smaller than the one of the incumbent may be foreclosed from the market (cf. Laffont et al, 1998-b and Lopez and Rey, 2009);
  - price differentiation may be abusively used by dominant operators in order to restrict the volume of off-net calls and to make small operators less attractive (cf. Hoernig, 2007);
  - in a competitive equilibrium, while the on-net price is set at its optimal level, the off-net price is set above its optimal level (cf. Berger, 2005 and Hoernig, 2007 for models with two operators and Hoernig, 2009 for a model with multiple operators);

- in the presence of intra-operator calling clubs, i.e. when a given user's friends and family mostly belong to the same network, an on-net/off-net discrimination gives rise to inefficient pricing by a new entrant (cf. Gabrielsen and Vagstad, 2008);
- the on-net/off-net differentiation hinders competition by creating an entry barrier for small operators who cannot viably replicate the price structure of incumbents (cf. Harbord and Pagnozzi, 2010).

Either because they are based on too particular assumptions, or, on the contrary, because they use a too general and therefore complex framework, the existing models do not yield a comprehensive analysis of the impact of on-net/off-net differentiation on the social welfare and its components. For instance, Hoernig (2009) does not come to a definite conclusion and states that the optimal pricing profile, either uniform or discriminatory, likely depends on the characteristics of the demand function.

The aim of the model developed here is to provide a complete toolkit to analyze the social welfare and its distribution between operators and consumers in the case where linearity of traffic demand functions is a sensible assumption. Similarly to Hoernig (2009), our model allows for an arbitrary number of operators, accounts for asymmetries of costs and market shares, as well as incorporates a call externality (utility of received calls). The main difference lies in the fact that subscription choices and hence operators' market shares are endogeneous in the Hoernig's model while they are exogeneous in our model. More precisely, we consider that the market is rigid and not completely transparent; consequently, consumers with bounded rationality choose their network on the basis of a number of criteria not limited to prices, resulting in inelastic market shares. Within this rigid structure of customer bases, consumers' demands for traffic in different directions depend on prices per minute. The linear form of the demand functions yields an easy computation of profits and surpluses, allowing us to compare the impact of different tariff structures on welfare and its components and, finally, to analyze the "tensions" that potentially generate migration flows between operators and thus a change in market shares.

Section 2 presents the model, its solution and theoretical impact of price changes on the welfare. Section 3 presents the results of the model's calibration using the data from the French mobile market and estimates the results of on-net and all-net price changes. Finally, Section 4 summarizes our main conclusions.

2 Theoretical model

This section presents the theoretical model. The subsection 2.1 presents the assumptions of the model and derives traffic demand functions for each operator in each traffic direction, subsections 2.3 and 2.4 study the social optimum and the market equilibrium respectively, subsection 2.5 assesses the impact on social welfare and its components of "neutral" price changes.

2.1 Demand modeling

2.1.1 Customer base, prices and traffic

We consider a national mobile market of the size M divided between n operators ($i = 1, 2, \ldots, n$), each serving M_i clients: $\sum_{i=1}^{n} M_i = M$. In the absence of a major price change or a sudden entry, market shares M_i/M are assumed to be stable and independent of minute prices. We consider that the market is rigid and not completely transparent; consequently, consumers with bounded rationality choose their network on the basis of a number of criteria not limited to prices, resulting in inelastic market shares. Such was for instance the case in the Metropolitan French mobile market from the end of 1990s and before the arrival of the fourth operator Free in 2012: market shares were stable over time with only small fluctuations.

We assume that an operator may set different prices for a minute of voice communication depending on the call recipient's network. Operator i sets price p_i^j for a minute of call from network i to network j, so that $\mathbf{p}_i = [p_i^1, p_i^2, \ldots, p_i^n]$ is the row vector of prices set by operator i.

Let Q_i^j denote the traffic in minutes originated in network i and addressed to network j, so that $\mathbf{Q}_i = [Q_i^1, Q_i^2, \ldots, Q_i^n]'$ is the column vector of traffic volumes sent from network i.[1] Similarly, let $\mathbf{Q}^i = [Q_1^i, Q_2^i, \ldots, Q_n^i]'$ denote the column vector of traffic volumes sent to network i.

In a stylized manner, all takes place as though the customer base of operator i was composed of M_i identical i-representatives, each of them generating the traffic q_i^j in the traffic direction j:

$$q_i^j = \frac{Q_i^j}{M_i}, \quad \mathbf{q}_i = [q_i^1, q_i^2, \ldots, q_i^n]' = \frac{\mathbf{Q}_i}{M_i}.$$

2.1.2 Individual preferences

The maximal volume of traffic that an individual would generate in a given traffic direction under a zero price is assumed to be proportional to the

[1] The symbol "$'$" designates matrix transposition.

number of potential recipients in this direction: the ceiling level of individual traffic increases with the number of parties to whom this traffic may be addressed. Hence, the ceiling level of an i-representative in direction j equates $\sigma_i M_j$, where the *communication potential* $\sigma_i > 0$ is higher for those consumers who are more inclined to communicate.[2] Our model of individual preferences is therefore *gravitational*: a large subscriber base "attracts" more traffic than a small one.

The gravitational core will now be extended in four ways in order to account for a *club effect*, a *saturation effect*, an *effect of substitution* between traffic directions, and a *call externality* .

1. *Club effect*. The "calling club" of a given individual is not uniformly distributed over the clients of different operators. This heterogeneous distribution is influenced by the interdependence of individual subscription choices, all the more as it is encouraged by the "*friends and family*" commercial discounts often introduced by operators. To take this into account, we assume that (excluding saturation and substitution effects) the traffic ceiling of an i-representative in direction j is not $\sigma_i M_j$ but $\sigma_i M_{j/i}$, where

$$M_{j/i} = \omega_{j/i} M_j, \quad \omega_{i/i} \geq 1, \quad \omega_{j/i} \leq 1, \quad \sum_{j=1}^{n} M_{j/i} = M.$$

$M_{j/i}$ is interpreted as the *apparent size* of network j as it is perceived by an i-representative, and parameter $\omega_{j/i}$ will be called the *shaping coefficient*.

2. *Saturation effect*. Like most consumer goods, mobile telephony is subject to saturation of consumption. We thus assume that the marginal utility of a minute of communication decreases linearly with the volume of traffic. So (not accounting for the substitution effect), the utility $\partial w_i / \partial q_i^j$ that an i-representative obtains from an additional minute of traffic in direction j writes

$$\frac{\partial w_i}{\partial q_i^j} = v_i M_{j/i}^{-2a} \left(M_{j/i} - \frac{q_i^j}{\sigma_i} \right).$$

Parameter v_i measures the *psychological value* of communication time for an i-representative, while parameter $a \in [0, 1]$ is an inverse measure of her/his *appetite for the number of called parties*.

The utility of the "first" minute of communication in traffic direction j is equal to $v_i M_{j/i}^{1-2a}$. When $a = 0$, the consumer is fully sensitive to the availability of a large base of callees $M_{j/i}$ and she/he is indifferent to a small

[2]Parameter σ_i is interpreted as an i-representative's maximal traffic volume issued towards a normalized "club" of size 1 million users; σ_i has the physical dimension of a duration and is expressed in minutes.

average duration per callee, $1/M_{j/i}$. When $a = 1/2$, the utility v_i of the first minute is independent of the number of potential callees; it is thus the same for each traffic direction j. When $a = 1$, the consumer is fully sensitive to a long average communication time per callee $1/M_{j/i}$ and she/he is indifferent to a small base $M_{j/i}$ of callees.

The utility of the "last" minute, the one that makes the customer reach the traffic ceiling $\sigma_i M_{j/i}$ in direction j, is zero. So, $\sigma_i M_{j/i}$ is interpreted as the consumption of saturation in direction j by an i-representative, i.e. the level of traffic q_i^j that achieves the maximum of individual utility w_i.

3. *Substitution effect.* With a consumer being able, up to a certain extent, to shift minutes from one traffic direction to another, the utility of one additional minute in a given direction depends not only on the traffic volume in this direction, but also on the traffic volumes in other directions.

Including the effects of substitution between traffic directions, the marginal utility of making calls finally writes

$$\frac{\partial w_i}{\partial q_i^j} = v_i \sum_{k=1}^{n} \frac{\xi_i^{j,k}}{M_{j/i}^a M_{k/i}^a} \left(M_{k/i} - \frac{q_i^k}{\sigma_i} \right), \tag{1}$$

where coefficients $\xi_i^{j,k}$ measure the intensity of substitution between directions j and k for an i-representative. These coefficients are assumed to be symmetric and to comply with the following conditions:

$$\xi_i^{j,j} = 1, \quad k \neq j : 0 \leq \xi_i^{j,k} = \xi_i^{k,j} < 1.$$

4. *Call externality.* A consumer derives utility not only from calls she/he makes but also from calls she/he recieves. We assume that an i-representative who receives Q_j^i/M_i minutes from network j obtains the individual benefit

$$r_j^i = \rho_i v_i M_{j/i}^{1-2a} \frac{Q_j^i}{M_i}.$$

Parameter ρ_i measures the relative intensity of the call externality. More exactly (not accounting for the saturation and substitution effects), the utility derived by an i-representative from a minute sent to network j is equal to $v_i M_{j/i}^{1-2a}$, so that ρ_i is the ratio between the utility of a minute received and the utility of a minute sent.

2.1.3 Utility function

Let us introduce the following notations:

$$\mathbf{X}_i = \mathbf{X}'_i = \begin{pmatrix} 1 & \xi_i^{2,1} & \cdots & \xi_i^{n-1,1} & \xi_i^{n,1} \\ \xi_i^{1,2} & 1 & \cdots & \xi_i^{n-1,2} & \xi_i^{n,2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \xi_i^{1,n-1} & \xi_i^{2,n-1} & \cdots & 1 & \xi_i^{n,n-1} \\ \xi_i^{1,n} & \xi_i^{2,n} & \cdots & \xi_i^{n-1,n} & 1 \end{pmatrix},$$

$$\mathbf{M}_{\cdot/i} = \mathrm{Diag}[M_{1/i}, M_{2/i}, ..., M_{n/i}],$$

$$\mathbf{X}_i(a) = \mathbf{M}_{\cdot/i}^{-a} \cdot \mathbf{X}_i \cdot \mathbf{M}_{\cdot/i}^{-a},$$

$$\boldsymbol{\nabla}_{\mathbf{q}_i} = \left[\frac{\partial}{\partial q_i^1}, \frac{\partial}{\partial q_i^2}, \cdots, \frac{\partial}{\partial q_i^n}\right],$$

where " · " denotes the matrix product and "Diag" denotes a diagonal matrix. The symmetric matrix $\mathbf{X}_i$ of substitution coefficients has positive component.[3] and is invertible.

Integrating the above specification of marginal utilities (1) with respect to outgoing traffic volumes $\mathbf{q}_i$, we obtain a quadratic expression of utility w_i of an i-representative. Summing over all M_i users of network i, we obtain the aggregate utility W_i that the clients of operator i derive from their outgoing calls:

$$W_i = M_i u_i + v_i \left(\mathbf{1} \cdot \mathbf{M}_{\cdot/i} - \frac{\mathbf{Q}'_i}{2\sigma_i M_i}\right) \cdot \mathbf{X}_i(a) \cdot \mathbf{Q}_i, \tag{2}$$

where u_i is a constant

The utility W_i of outgoing traffic is complemented by the external benefit R^i from received traffic:

$$R^i = \rho_i v_i \mathbf{1} \cdot \mathbf{M}_{\cdot/i}^{1-2a} \cdot \mathbf{Q}^i. \tag{3}$$

The total utility $W_i + R^i$ of the clients of operator i is the sum of three components: a fixed component $M_i u_i$ independent of outgoing and incoming traffic volumes; an "internal" component $W_i - M_i u_i$, depending on traffic volumes Q_i^j addressed to each destination network $j = 1, 2, ..., n$; and an "external" component R^i depending on traffic volumes Q_j^i received from each originating network $j = 1, 2, ..., n$.

[3]thus is positively defined on $\mathbb{R}_+^n$.

2.1.4 Demand functions

Assume that incoming calls are not charged. Also consider that each mobile operator i proposes a menu of different packages with tariffs that are linearly progressive with respect to the maximal volume of traffic offered in a package. Thus, the bill ϕ_i of an i-representative, who sends q_i^j minutes towards networks $j = 1, 2, ..., n$, may be approached by an affine function:

$$\phi_i = f_i + \sum_{j=1}^{n} p_i^j q_i^j = f_i + \mathbf{p}_i \cdot \mathbf{q}_i,$$

where f_i is a fixed component common to all packages of operator i, and p_i^j is the price of a minute from i to j.

Such an affine pricing is equivalent to a package pricing as regards the program of consumer surplus maximization: indeed, under a package pricing, the Lagrange multiplier of the volume constraint plays exactly the same role as the price of a minute under an affine tariff. Hence, to each package we may associate an equivalent affine tariff, generating the same traffic demand, the same bill and the same consumer surplus.

By aggregation, the retail revenues of operator i write

$$\Phi_i = M_i f_i + \sum_{j=1}^{n} p_i^j Q_i^j = M_i f_i + \mathbf{p}_i \cdot \mathbf{Q}_i. \tag{4}$$

The consumption surplus S_i that clients of i obtain from their outgoing traffic is then derived from (2) and (4):

$$\begin{aligned} S_i &= W_i - \Phi_i \\ &= M_i(u_i - f_i) + [v_i \mathbf{1} \cdot \mathbf{M}_{./i} \cdot \mathbf{X}_i(a) - \mathbf{p}_i] \cdot \mathbf{Q}_i - \frac{v_i}{2\sigma_i M_i} \mathbf{Q}_i' \cdot \mathbf{X}_i(a) \cdot \mathbf{Q}_i \ . \end{aligned} \tag{5}$$

Surplus S_i being a concave quadratic function of outgoing communication volumes Q_i^j, its maximum is fully determined by first-order conditions.[4] These lead to the inverse system of demand:

$$\mathbf{p}_i = \boldsymbol{\pi}_i(\mathbf{Q}_i) = v_i \left(\mathbf{1} \cdot \mathbf{M}_{./i} - \frac{\mathbf{Q}_i'}{\sigma_i M_i} \right) \cdot \mathbf{X}_i(a), \tag{6}$$

[4] S_i is concave because matrix $\mathbf{X}_i(a)$ is positively defined on $\mathbb{R}_+^n$. Its maximum is determined by first-order conditions under the hypothesis that the optimum is interior, i.e. not achieved on the frontier of $\mathbb{R}_+^n$.

from which we compute the direct demand functions:

$$\mathbf{Q}_i = \mathbf{D}_i(\mathbf{p}_i) = \sigma_i M_i \left(\mathbf{M}_{\cdot/i} \cdot \mathbf{1}' - \mathbf{A}_i(a) \cdot \mathbf{p}_i'\right), \tag{7}$$

where we have introduced the symmetric matrix $\mathbf{A}_i(a)$:

$$\mathbf{A}_i(a) = \mathbf{A}_i'(a) = \frac{\mathbf{X}_i^{-1}(a)}{v_i} = \mathbf{M}^a_{\cdot/i} \cdot \mathbf{A}_i \cdot \mathbf{M}^a_{\cdot/i},$$

$$\mathbf{A}_i = \mathbf{A}_i' = \frac{\mathbf{X}_i^{-1}}{v_i} = \begin{pmatrix} \alpha_i^1 & -\beta_i^{1,2} & \cdots & -\beta_i^{1,n-1} & -\beta_i^{1,n} \\ -\beta_i^{2,1} & \alpha_i^1 & \cdots & -\beta_i^{2,n-1} & -\beta_i^{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\beta_i^{n-1,1} & -\beta_i^{n-1,2} & \cdots & \alpha_i^{n-1} & -\beta_i^{n-1,n} \\ -\beta_i^{n,1} & -\beta_i^{n,2} & \cdots & -\beta_i^{n,n-1} & \alpha_i^n \end{pmatrix}.$$

In the following we shall assume that coefficients α_i^j and $\beta_i^{j,k}$ satisfy conditions[5]

$$\alpha_i^j > 0, \quad j \neq k : \beta_i^{j,k} = \beta_i^{k,j} \geq 0,$$
$$\widetilde{\alpha}_i^j = \alpha_i^j - \sum_{k \neq j} \left(\frac{M_{k/i}}{M_{j/i}}\right)^a \beta_i^{j,k} > 0.$$

Sensitivity of the traffic demand Q_i^j to an absolute variation in price p_i^k is constant, i.e. independent of prices and traffic volumes.[6] The on-net traffic of a given operator and each of the off-net traffics towards other operators are substitutes.

From the demand function (7) we obtain $\widehat{\mathbf{Q}}_\mathbf{i}^\mathbf{j}$, the traffic matrix in the saturation point that reflects the traffic pattern under zero prices:

$$\widehat{\mathbf{Q}}_\mathbf{i}^\mathbf{j} = \sigma_i M_i M_{j/i} = \sigma_i \omega_{j/i} M_i M_j.$$

If all operators had the same clients' profile ($\forall i \ \sigma_i = \sigma$) and club effects were negligible ($\omega_{j/i} = 1$), then the saturation matrix would be both symmetric and gravitational, its elements being proportional to the product of the mass M_i of the origination network and the mass M_j of the destination network.

[5]These conditions guarantee that both $\mathbf{X}_i(a)$ and $\mathbf{A}_i(a)$ are invertible and positively defined.

[6]As a result, price elasticities are variable, i.e. proportional to prices and inversely proportional to quantities.

2.2 Social optimum

Let c_i (resp. c^i) denote the marginal cost of carrying a minute of outgoing traffic (resp. incoming traffic) on the network of operator i. Let g_i denote the non-traffic-sensitive cost per subscriber; the global cost incurred by operator i (excluding traffic termination payments) is then given by

$$C_i = M_i g_i + \mathbf{1} \cdot (c_i \mathbf{Q}_i + c^i \mathbf{Q}^i). \tag{8}$$

From (2), (3) and (8) we derive the total welfare Σ:

$$\begin{aligned} \Sigma &= \sum_{i=1}^{n} (W_i + R^i - C_i) \\ &= \sum_{i=1}^{n} M_i(u_i - g_i) + \sum_{i=1}^{n} v_i \left[\mathbf{1} \cdot \mathbf{M}_{\cdot/i} - \frac{\mathbf{Q}_i'}{2\sigma_i M_i} \right] \cdot \mathbf{X}_i(a) \cdot \mathbf{Q}_i \\ &\quad + \mathbf{1} \cdot \sum_{i=1}^{n} \left[(\rho_i v_i \mathbf{M}_{\cdot/i}^{1-2a} - c^i) \cdot \mathbf{Q}^i - c_i \mathbf{Q}_i \right]. \end{aligned} \tag{9}$$

The function Σ being quadratic and concave with respect to traffic volumes Q_i^j,[7] the social optimum is characterized by first-order conditions. By introducing notations

$$\mathbf{r}_i = [\rho_1 v_1 M_{i/1}^{1-2a}, \rho_2 v_2 M_{i/2}^{1-2a}, ..., \rho_n v_n M_{i/n}^{1-2a}] \quad \text{and} \quad \mathbf{c}^{\cdot} = [c^1, c^2, ..., c^n],$$

from first-order conditions and from the expression of inverse demand functions (6), optimal prices are

$$\mathbf{p}_i^* = c_i \mathbf{1} + \mathbf{c}^{\cdot} - \mathbf{r}_i,$$

and, from (7), optimal traffic volumes write

$$\mathbf{Q}_i^* = \sigma_i M_i \left(\mathbf{M}_{\cdot/i} \cdot \mathbf{1}' - \mathbf{A}_i(a) \cdot \mathbf{p}_i^{*\prime} \right).$$

The optimal price p_i^{j*} in direction j and the differential $p_i^{j*} - p_i^{i*}$ between the off-net price and the on-net price are

$$\begin{aligned} p_i^{j*} &= c_i + c^j - \rho_j v_j M_{i/j}^{1-2a}, \\ p_i^{j*} - p_i^{i*} &= c^j - c^i - (\rho_j v_j M_{i/j}^{1-2a} - \rho_i v_i M_{i/i}^{1-2a}). \end{aligned}$$

[7] because matrix $\mathbf{X}_i(a)$ is positively defined on $\mathbb{R}_+^n$.

Proposition 1 *At the social optimum, the price of a minute in a given direction is equal to the associated marginal cost minus the call externality derived from the reception of this minute.*[8]

Corollary 1 *The efficient off-net/on-net price differentiation is equal to the differential between the marginal termination cost in the off-net direction and the marginal termination cost in the on-net direction, corrected by the differential in the utility of receiving a minute from the off-net or the on-net direction. Therefore, if the marginal cost differential is weak or inexistant and if call externalities are zero, any significant price difference between on-net and off-net directions is inefficient.*

2.3 "Monopolistic" equilibrium

With the subscriber base of each operator taken as exogeneous, each operator i holds a monopoly in the retail sub-market of traffic originated on its network. It follows that in our model the oligopoly is of a "monopolistic" type. Termination rates are the charges that one telecommunications operator charges to another for terminating calls on its network.

2.3.1 Unregulated equilibrium

In the absence of regulation of termination rates, the equilibrium is the solution of a two-step dynamic game:

- in the first step, each operator i sets its call termination rate τ^i by maximizing the interconnection profit generated by incoming traffic, the termination rates of other operators being considered as given;
- in the second step, each operator i sets its retail prices p_i^j in each direction $j = 1, 2, ..., n$ by maximizing its retail profit at given termination rates.

[8] However, in practice, substracting the call externality $\rho_j v_j M_{i/j}$ when setting the retail price p_i^j makes the minute of outgoing traffic unprofitable ($p_i^{j*} < c_i + c^j$). This deficit may be compensated *via* charging received calls in the retail market, thus allowing for internalization of the externality by operators. If the retail price of a minute from network j to network i were set equal to its marginal utility $\rho_j v_j M_{i/j}^{1-2a}$, then consumers would not care about the volumes they receive, since their marginal surplus from a minute received would be zero. The demands for outgoing calls would not change, the optimum would be preserved and the internalisation would be a pure transfer: consumers would loose the benefit from the call externality while operators would compensate the marginal deficit of an outgoing minute.

This game is solved by backward induction, going upstream from the retail market to the wholesale market.

1. Profit maximization in the retail market.

Let γ_i^j be the marginal "accounting" cost incurred by operator i per minute from i to j:

$$\gamma_i^j = \begin{cases} c_i + c^i, & \text{if } \ j = i, \\ c_i + \tau^j, & \text{if } \ j \neq i. \end{cases}$$

Let

$$\boldsymbol{\gamma}_i = [\gamma_i^1, \gamma_i^2, ..., \gamma_i^n]$$

be the vector of "accounting" costs. From (4), the profit of operator i in the retail market writes

$$\begin{aligned} \Pi_i &= \Phi_i - \gamma_i \cdot \mathbf{Q}_i = M_i f_i + (\mathbf{p}_i - \boldsymbol{\gamma}_i) \cdot \mathbf{Q}_i \\ &= M_i f_i + v_i \left(\mathbf{1} \cdot \mathbf{M}_{\cdot/i} - \frac{\mathbf{Q}_i'}{\sigma_i M_i} \right) \cdot \mathbf{X}_i(a) \cdot \mathbf{Q}_i - \boldsymbol{\gamma}_i \cdot \mathbf{Q}_i. \end{aligned} \tag{10}$$

This profit is maximized when marginal revenues equate marginal costs, leading to prices and traffic volumes at equilibrium:

$$\begin{aligned} \overline{\mathbf{p}}_i &= \frac{1}{2} \left[\gamma_i + v_i \mathbf{1} \cdot \mathbf{M}_{\cdot/i} \cdot \mathbf{X}_i(a) \right] \\ &= \mathbf{p}_i^* + \frac{1}{2} \left[v_i \mathbf{1} \cdot \mathbf{M}_{\cdot/i} \cdot \mathbf{X}_i(a) - \boldsymbol{\gamma}_i \right] + (\boldsymbol{\gamma}_i - c_i \mathbf{1} - \mathbf{c}^{\cdot}) + \mathbf{r}_i, \\ \overline{\mathbf{Q}}_i &= \sigma_i M_i \left[\mathbf{M}_{\cdot/i} \cdot \mathbf{1}' - \mathbf{A}_i(a) \cdot \overline{\mathbf{p}}_i' \right] = \frac{\sigma_i M_i}{2} \left[\mathbf{M}_{\cdot/i} \cdot \mathbf{1}' - \mathbf{A}_i(a) \cdot \boldsymbol{\gamma}_i' \right]. \end{aligned}$$

The equilibrium price $\overline{p}_i^j$ in direction j and the differential $\overline{p}_i^j - \overline{p}_i^i$ between the off-net price and the on-net price are:

$$\begin{aligned} \overline{p}_i^j &= \frac{1}{2} \left(\gamma_i^j + v_i \sum_{k=1}^{n} M_{j/i}^{-a} M_{k/i}^{1-a} \xi_i^{j,k} \right), \\ \overline{p}_i^j - \overline{p}_i^i &= \frac{1}{2} (\tau^j - c^i) + \frac{v_i}{2} \sum_{k \neq i,j}^{n} \left(M_{j/i}^{-a} \xi_i^{j,k} - M_{i/i}^{-a} \xi_i^{i,k} \right) M_{k/i}^{1-a}. \end{aligned}$$

Proposition 2 *Prices at competitive equilibrium $\overline{p}_i^j$ are higher than socially optimal prices p_i^{j*} for three cumulative reasons. First, prices $\overline{p}_i^j$ incorporate a markup above the associated marginal cost γ_i^j ($v_i \mathbf{1} \cdot \mathbf{M}_{\cdot/i} \cdot \mathbf{X}_i(a) > \boldsymbol{\gamma}_i$). Second, margins paid on the wholesale markets are passed to the retail market ($\boldsymbol{\gamma}_i > c_i \mathbf{1} - \mathbf{c}^{\cdot}$). Finally, prices $\overline{p}_i^j$ ignore the call externality $\rho_j v_j M_{i/j}^{1-2a}$ ($\mathbf{r}_i > \mathbf{0}$).*

Corollary 2 *The competitive equilibrium leads to an off-net/on-net price differentiation since the differential between the termination price paid in a given off-net direction and the cost of an on-net minute ($\tau^j - c^i > 0$) tends to increase the off-net prices. In addition, depending on whether off-net/off-net substitution is more intensive or less intensive than the on-net/off-net substitution (i.e. whether $M_{j/i}^{-a}\xi_i^{j,k} > M_{i/i}^{-a}\xi_i^{i,k}$ or $M_{j/i}^{-a}\xi_i^{j,k} < M_{i/i}^{-a}\xi_i^{i,k}$), the off-net/on-net price gap is increased or decreased.*

2. Profit maximization in the wholesale market.
The interconnection profit of operator i writes

$$\Psi^i = (\tau^i - c^i)\mathbf{1} \cdot \overline{\mathbf{Q}}^i$$
$$= \frac{1}{2}(\tau^i - c^i)\sum_{j\neq i}\sigma_j M_j \left[M_{i/j} - M_{j/i}^{2a}\widetilde{\alpha}_j^i c_j + \sum_{k\neq i} M_{j/i}^a M_{k/i}^a \beta_j^{i,k}\tau^k - M_{j/i}^{2a}\alpha_j^i \tau^i \right]. \tag{11}$$

The maximization of Ψ^i with respect to τ^i, at all τ^k being given for $k \neq i$, yields the best response function $\tau^i(\boldsymbol{\tau}^{-i})$ of operator i:

$$\tau^i(\boldsymbol{\tau}^{-i}) = \frac{\sum_{k\neq i}\left(\sum_{j\neq i}\sigma_j M_j (M_{i/k}/M_{i/j})^a M \beta_j^{i,k}\right)\tau^k}{2\sum_{j\neq i}\sigma_j M_j \alpha_j^i} + \frac{\sum_{j\neq i}\sigma_j M_j\left(M_{i/j}^{1-2a} + \alpha_j^i c^i - \widetilde{\alpha}_j^i c_j\right)}{2\sum_{j\neq i}\sigma_j M_j \alpha_j^i}.$$

In the symmetric case (where costs per minute are the same for all networks, all operators have the same market share and the same preference function of representative consumer) and if club effects are absent and substitution effects are uniform, then the call termination rate is set at the same level $\overline{\tau}$ by each operator, according to the best response function:

$$c_i = c_{\cdot}\ ,\ c^i = c^{\cdot}\ ,\ M_i = M_{i/\cdot} = \frac{M}{n}\ ,\ \alpha_i^j = \alpha\ ,\ \beta_i^{j,k} = \beta\ \Rightarrow$$
$$\overline{\tau}^i = \overline{\tau} = c^{\cdot} + \frac{(M/n)^{1-2a} - [\alpha - (n-1)\beta](c^{\cdot} + c_{\cdot})}{2\alpha - (n-1)\beta}.$$

Proposition 3 *The strategic behavior of operators in the wholesale interconnection market leads each of them to set their termination rate above marginal cost. Since this markup is passed on to the off-net retail price (cf. supra), double marginalization occurs in the absence of wholesale price regulation.*

2.3.2 Regulated equilibrium

In the regulated equilibrium, the regulator sets price caps on termination rates (as it is done in Europe):

$$\tau^i \leq \mu^i + c^i.$$

The constraint is binding for each operator, so that

$$\boldsymbol{\gamma}_i = c_i\mathbf{1} + \mathbf{c}^{\cdot} + \boldsymbol{\mu}^{\cdot} \Leftrightarrow \gamma_i^j = c_i + c^j + \mu^j.$$

Consequently, the monopolistic equilibrium in the retail market leads to the following price system:

$$\begin{aligned} \overline{\mathbf{p}}_i &= \frac{1}{2}\left[c_i\mathbf{1} + \mathbf{c}^{\cdot} + \boldsymbol{\mu}^{\cdot} + v_i\mathbf{1}\cdot\mathbf{M}_{./i}\cdot\mathbf{X}_i(a)\right] \Leftrightarrow \\ \overline{p}_i^j &= \frac{1}{2}\left[c_i + c^j + \mu^j + v_i\left(\sum_{k=1}^{n} M_{j/i}^{-a} M_{k/i}^{1-a}\xi_i^{j,k}\right)\right], \end{aligned}$$

from which we get the differential between off-net and on-net prices:

$$j \neq i : \overline{p}_i^j - \overline{p}_i^i = \frac{c^j - c^i}{2} + \frac{v_i}{2}\sum_{k=1}^{n}(M_{j/i}^{-a}\xi_i^{j,k} - M_{i/i}^{-a}\xi_i^{i,k})M_{k/i}^{1-a} + \frac{\mu^j}{2}.$$

Proposition 4 *Since the wholesale markup is passed on to the retail market, a cap on termination rates gives the operators an incentive to set the differential between off-net and on-net prices above the level justified by the marginal cost differential and the market share differential. However, this undesirable incentive gradually tends to disappear as the price-cap converges to the level of marginal costs ($\boldsymbol{\mu}^{\cdot} \to \mathbf{0}$).*

2.4 Impact of price changes

In this section, we study the impact of price changes on the mobile telephony market. These changes occur in some initial market state characterized by tariffs $(f_i, \mathbf{p}_i)$ and associated traffic volumes $\mathbf{Q}_i$. In the neighborhood of this state of reference, we consider *neutral* price changes $(\Delta f_i, \Delta \mathbf{p}_i)$, i.e. changes that do not modify consumers' spendings ($\Delta \Phi_i = 0$, cf. (4)):

$$\begin{aligned}\Delta f_i &= -\frac{1}{M_i}\Delta(\mathbf{p}_i \cdot \mathbf{Q}_i) \\ &= -\sigma_i \left[\mathbf{1} \cdot \mathbf{M}_{./i} - 2\mathbf{p}_i \cdot \mathbf{A}_i(a)\right] \cdot \Delta \mathbf{p}_i^{'} - \sigma_i \Delta \mathbf{p}_i \cdot \mathbf{A}_i(a) \cdot \Delta \mathbf{p}_i^{'}.\end{aligned}$$

A particular change made by operator i is completely specified by the vector $\Delta \mathbf{p}_i$ of the per-minute price variations in all traffic directions.

2.4.1 Impact on traffic volumes

With each operator holding a monopoly on the traffic originated in its own network, traffic volumes $\mathbf{Q}_i$ issued on network i are sensitive only to prices $\mathbf{p}_i$. From the expression of aggregate demand functions (7) we have

$$\Delta \mathbf{Q}_i = -\sigma_i M_i \mathbf{A}_i(a) \cdot \Delta \mathbf{p}_i', \tag{12}$$

and the impact on the total outgoing traffic Q_i of operator i is $\Delta Q_i = \mathbf{1} \cdot \Delta \mathbf{Q}_i$.

From these general expressions, we can compare the effects of on-net price cuts $\Delta^{\text{on}} \mathbf{p}_i = \Delta p_{+i} \mathbf{1}_i$ ($\Delta p_{+i} < 0$, $\mathbf{1}_i = (0, ..., 0, 1, 0, ..., 0)$) and all-net price cuts $\Delta^{\text{all}} \mathbf{p}_i = \Delta p_i \mathbf{1}$ ($\Delta p_i < 0$).

On-net price cuts:

$$\begin{aligned}\Delta^{\text{on}} Q_{+i} &= -\sigma_i M_i M_{i/i}^{2a} \alpha_i^i \Delta p_{+i} > 0, \\ j \neq i : \Delta^{\text{on}} Q_i^j &= \sigma_i M_i M_{i/i}^a M_{j/i}^a \beta_i^{i,j} \Delta p_{+i} \leq 0, \\ \Delta^{\text{on}} Q_i &= -\sigma_i M_i M_{i/i}^{2a} \widetilde{\alpha}_i^i \Delta p_{+i} > 0.\end{aligned}$$

All-net price cuts:

$$\begin{aligned}\forall j : \Delta^{\text{all}} Q_i^j &= -\sigma_i M_i M_{j/i}^{2a} \widetilde{\alpha}_i^j \Delta p_i > 0, \\ \Delta^{\text{all}} Q_i &= -\sigma_i M_i \left(\sum_{j=1}^{n} M_{j/i}^{2a} \widetilde{\alpha}_i^j\right) \Delta p_i > 0.\end{aligned}$$

Proposition 5 *An on-net price cut leads to an increase in the on-net traffic (own-elasticity effect) and to a decrease in the off-net traffic (cross-elasticity substitution effect). Both on-net and all-net price cuts lead to an increase in the total outgoing traffic of the operator who implements it. This increase in the total traffic is stronger (resp. weaker) under an all-net price cut than under an on-net price cut if the ratio of the latter to the former does not* *exceed (resp. exceeds) a threshold δ_i^Q which depends on the coefficients of substitutability:*

$$\frac{\Delta^{all} Q_i}{\Delta^{on} Q_i} \gtreqless 1 \Leftrightarrow \frac{|\Delta p_{+i}|}{|\Delta p_i|} \lesseqgtr \delta_i^Q = 1 + \frac{1}{M_{i/i}^{2a} \widetilde{\alpha}_i^i} \sum_{j \neq i} M_{j/i}^{2a} \widetilde{\alpha}_i^j.$$

When $|\Delta p_{+i}| = \delta_i^Q |\Delta p_i|$, on-net and all-net price cuts are equivalent in terms of outgoing traffic.

2.4.2 Impact on consumers

With the bill remaining the same after a neutral price change, the variation of consumers' surplus S_i in network i (excluding the call externality) coincides with the variation of the utility W_i derived from the issued traffic (see (2) and (5)). The latter being quadratic with respect to traffic volumes and, consequently, with respect to per-minute prices, the variation ΔS_i writes

$$\Delta S_i = \Delta W_i = \boldsymbol{\nabla}_{\mathbf{Q}_i} W_i \cdot \Delta \mathbf{Q}_i + \frac{1}{2} \Delta \mathbf{Q}_i' \cdot \boldsymbol{\nabla}^2_{\mathbf{Q}_i^2} W_i \cdot \Delta \mathbf{Q}_i. \tag{13}$$

As we have

$$\boldsymbol{\nabla}_{\mathbf{Q}_i} W_i = \mathbf{p}_i, \quad \boldsymbol{\nabla}^2_{\mathbf{Q}_i^2} W_i = \boldsymbol{\nabla}_{\mathbf{Q_i}} \mathbf{p}_i = -\frac{v_i \mathbf{X}_i(a)}{\sigma_i M_i} = -\frac{\mathbf{A}_i^{-1}(a)}{\sigma_i M_i},$$

and (12),we finally obtain

$$\Delta S_i = -\sigma_i M_i \left(\mathbf{p}_i + \frac{1}{2} \Delta \mathbf{p}_i \right) \cdot \mathbf{A}_i(a) \cdot \Delta \mathbf{p}_i'.$$

Note that the first order approximation yields

$$\Delta S_i \simeq \mathbf{p}_i \cdot \Delta \mathbf{Q}_i = -\sigma_i M_i \mathbf{p}_i \cdot \mathbf{A}_i(a) \cdot \Delta \mathbf{p}_i'.$$

Proposition 6 *The first-order impact of a neutral price change on the consumer's surplus is equal to the virtual variation of the bill at constant prices, i.e. to its "sub-variation" due to traffic variation only. Thus, the impact on surplus increases with the price-elasticity of demand (coefficients of price sensitivity α_i^j).*

Corollary 3 *If all operators simultaneously decide to implement "equivalent" neutral price changes, i.e. producing the same virtual variation of the bill for their respective customer bases ($j \neq i : \mathbf{p}_i \cdot \Delta \mathbf{Q}_i = \mathbf{p}_j \cdot \Delta \mathbf{Q}_j$), then this joint price change will not significantly change the situation of consumers, will not lead to significant customer migration flows and will therefore leave the market structure quasi-stable.*

Variation ΔS_i of surplus generated by the outgoing traffic is complemented by the variation ΔR^i of the external benefit from received calls (see (5)):

$$\Delta R^i = \rho_i v_i \mathbf{1} \cdot \mathbf{M}_{\cdot/i}^{1-2a} \cdot \Delta \mathbf{Q}^i. \tag{14}$$

Let us then compare the respective effects of simultaneous on-net price cuts and all-net price cuts.

On-net price cuts:

$$\Delta^{\text{on}} S_i = -\sigma_i M_i M_{i/i}^{2a} \left[\left(\alpha_i^i p_i^i - \sum_{j \neq i} (M_{j/i}/M_{i/i})^a \beta_i^{i,j} p_i^j \right) \Delta p_{+i} - \frac{\sigma_i \alpha_i^i}{2} (\Delta p_{+i})^2 \right],$$

$$\Delta^{\text{on}} R^i = -\rho_i v_i \sigma_i M_i M_{i/i} \alpha_i^i \Delta p_{+i} + \rho_i v_i \sum_{j \neq i} \sigma_j M_j M_{j/i}^{1-2a} M_{i/i}^a M_{j/j}^a \beta_j^{i,j} \Delta p_{+j}.$$

All-net price cuts:

$$\Delta^{\text{all}} S_i = -\sigma_i M_i \left(\sum_{j=1}^n M_{j/i}^{2a} \widetilde{\alpha}_i^j p_i^j \right) \Delta p_i - \frac{\sigma_i M_i}{2} \left(\sum_{j=1}^n M_{j/i}^{2a} \widetilde{\alpha}_i^j \right) (\Delta p_i)^2,$$

$$\Delta^{\text{all}} R^i = -\rho_i v_i \sum_{j=1}^n \sigma_j M_j M_{j/i}^{1-2a} M_{i/j}^{2a} \widetilde{\alpha}_j^i \Delta p_j.$$

Proposition 7 *At first-order approximation, an all-net price cut yields more (resp. less) consumer surplus than an on-net price cut if the ratio of the latter to the former does not exceed (resp. exceeds) a threshold δ_i^S that depends on coefficients of substitutability and prices:*

$$\frac{\Delta^{all} S_i}{\Delta^{on} S_i} \gtreqless 1 \Leftrightarrow \frac{|\Delta p_{+i}|}{|\Delta p_i|} \lesseqgtr \delta_i^S = \frac{\displaystyle\sum_{j=1}^n M_{j/i}^{2a} \widetilde{\alpha}_i^j p_i^j}{M_{i/i}^{2a} \left(\alpha_i^i p_i^i - \displaystyle\sum_{j \neq i} (M_{j/i}/M_{i/i})^a \beta_i^{i,j} p_i^j \right)}.$$

When $|\Delta p_{+i}| = \delta_i^S |\Delta p_i|$, the on-net and off-net price cuts are equivalent in terms of consumer surplus.

Proposition 8 *An on-net price cut leads to an increase in the external benefit of receiving calls for the clients of the operator who implements the price cut and it decreases this benefit for the clients of a third-party operator. An all-net price cut increases the benefit of receving calls for all the called parties, whether or not their operator is the one who implements the price cut.*

2.4.3 Impact on operators

With retail revenues remaining constant under a neutral price change, the profit variation on the retail market is reduced to the opposite of the cost variation. From (10) and (11), the respective variations of the retail profit Π_i and of the wholesale profit Ψ^i of operator i are

$$\Delta\Pi_i = -\gamma_i \cdot \Delta\mathbf{Q}_i = \sigma_i M_i \gamma_i \cdot \mathbf{A}_i(a) \cdot \Delta\mathbf{p}'_i, \tag{15}$$

$$\Delta\Psi^i = (\tau^i - c^i)\sum_{j\neq i}\Delta Q^i_j = -(\tau^i - c^i)\mathbf{1}_i \cdot \sum_{j\neq i}\sigma_j M_j \mathbf{A}_j(a) \cdot \Delta\mathbf{p}'_j. \tag{16}$$

In the case of simultaneous on-net or all-net price cuts, the impact on profits takes the following form.

On-net price cuts:

$$\Delta^{\text{on}}\Pi_i = \sigma_i M_i M^{2a}_{i/i}\left(\alpha^i_i\gamma^i_i - \sum_{j\neq i}(M_{j/i}/M_{i/i})^a\beta^{i,j}_i\gamma^j_i\right)\Delta p_{+i} < 0,$$
$$\Delta^{\text{on}}\Psi^i = (\tau^i - c^i)\sum_{j\neq i}\sigma_j M_j M^a_{i/j} M^a_{j/j}\beta^{i,j}_j\Delta p_{+j} < 0.$$

All-net price cuts:

$$\Delta^{\text{all}}\Pi_i = \sigma_i M_i\left(\sum_{j=1}^{n} M^{2a}_{j/i}\widetilde{\alpha}^j_i\gamma^j_i\right)\Delta p_i < 0,$$
$$\Delta^{\text{all}}\Psi^i = -(\tau^i - c^i)\sum_{j\neq i}\sigma_j M_j M^{2a}_{i/j}\widetilde{\alpha}^i_j\Delta p_j > 0.$$

Proposition 9 *A neutral price cut, either on-net or all-net, decreases the retail profit of the operator who implements it.*

An all-net price cut leads to a higher (resp. lower) decrease in the retail profit than an on-net price cut if the ratio of the latter to the former does not exceed (resp. exceedes) a threshold δ^Π_i *that depends on coefficients of substitutability and costs:*

$$\frac{\left|\Delta^{all}\Pi_i\right|}{\left|\Delta^{on}\Pi_i\right|} \gtreqless 1 \Leftrightarrow \frac{|\Delta p_{+i}|}{|\Delta p_i|} \lesseqgtr \delta^\Pi_i = \frac{\sum_{j=1}^{n} M^{2a}_{j/i}\widetilde{\alpha}^j_i\gamma^j_i}{M^{2a}_{i/i}\left(\alpha^i_i\gamma^i_i - \sum_{j\neq i}(M_{j/i}/M_{i/i})^a\beta^{i,j}_i\gamma^j_i\right)}.$$

When $|\Delta p_{+i}| = \delta^\Pi_i |\Delta p_i|$*, the on-net and off-net price cuts are equivalent in terms of retail profits.*

Proposition 10 *An on-net (resp. all-net) price cut made by a given operator negatively (resp. positively) affects interconnection profits of other operators since it leads to a decrease (resp. an increase) in the traffic issued towards each of off-net directions.*

As a result, for the same retail profit decrease, to cut all-net prices rather than on-net prices generates a substantial gain for each operator i:

$$\Delta(\Delta\Psi^i) = -(\tau^i - c^i)\sum_{j\neq i}\sigma_j M_j M_{i/j}^{2a}\left[\widetilde{\alpha}_j^i \Delta p_j + (M_{j/j}/M_{i/j})^a \beta_j^{i,j}\Delta p_{+j}\right] > 0.$$

Consequently, if on-net and all-net price cuts were adjusted so that they had the same impact on the consolidated profits of each operator (and not on their retail profits as assumed so far), then to decide an all-net instead of an on-net price cut would pass the gain to the customers of the different operators.

Corollary 4 *Consider two price cuts that produce the same effect on consolidated profits, where consolidated profits include retail profits from outgoing calls and interconnection profits from incoming calls. Then the operators transfer more consumer surplus to their customers under simultaneous all-net price cuts than under simultaneous on-net price cuts.*

2.4.4 Global impact

By cumulating impacts on the surplus of all the subscribers and on the profits of all the operators, i.e. summing up (13), (14), (15) and (16), we derive the impact on social welfare Σ:

$$\Delta\Sigma = \sum_{i=1}^{n}\left(\Delta S_i + \Delta R^i + \Delta\Pi_i + \Delta\Psi^i\right)$$

from which, after rearranging terms and introducing optimal prices $\mathbf{p}_i^*$, we obtain

$$\Delta\Sigma = \sum_{i=1}^{n}\sigma_i M_i(\mathbf{p}_i - \mathbf{p}_i^* - \frac{\Delta\mathbf{p}_i}{2})\cdot\mathbf{A}_i(a)\cdot\Delta\mathbf{p}_i'.$$

The first order variation of the social welfare is zero in the neighborhood of the social optimum ($\mathbf{p}_i = \mathbf{p}_i^*$).

Finally, examine the two particular cases of simultaneous on-net and simultaneous all-net price cuts.

On-net price cuts:

$$\Delta^{\text{on}}\Sigma = -\sum_{i=1}^{n}\sigma_i M_i M_{i/i}^{2a}\left[\alpha_i^i(p_i^i - p_i^{i*}) - \sum_{j\neq i}(M_{j/i}/M_{I/i})^a \beta_i^{i,j}(p_i^j - p_i^{j*})\right]\Delta p_{+i}$$
$$-\frac{1}{2}\sum_{i=1}^{n}\sigma_i M_i M_{i/i}^{2a}\alpha_i^i \Delta^2 p_{+i} > 0.$$

All-net price cuts:

$$\Delta^{\text{all}}\Sigma = -\sum_{i=1}^{n}\sigma_i M_i\left[\sum_{j=1}^{n} M_{j/i}^{2a}\widetilde{\alpha}_i^j(p_i^j - p_i^{j*})\right]\Delta p_i$$
$$+\frac{1}{2}\sum_{i=1}^{n}\sigma_i M_i\left[\sum_{j=1}^{n} M_{j/i}^{2a}\widetilde{\alpha}_i^j\right]\Delta^2 p_i > 0.$$

Proposition 11 *Both all-net and on-net price cuts lead to an increase in the social welfare, an increase in the consumers' surplus and a decrease in the retail profits of operators. However, an all-net price cut improves the interconnection balance of third party operators, whereas an on-net price cut worsens it. Similarly, an all-net price cut leads to an increase in the external benefit of calls received by the clients of third-party operators, whereas an on-net price cut leads to its decrease.*

3 Quantitative analysis

The objective of this section is to proceed to a numerical simulation of the effects of the neutral tariff cuts made by operators starting from an initial situation characterized by uniform all-net pricing. Two alternative price cuts will be evaluated and compared: on the one hand, an on-net cut corresponding to the introduction of packages with unlimited on-net calls; on the other hand, an all-net cut when the same price cuts are applied to all the traffic directions.

The reference market is the French mobile market in 2003, the year preceding introduction of important on-net price reduction. Publicly available numerical data, our main hypotheses and calibration procedure are described in the Appendix.

3.1 Remarkable states of the market

The calbrated model allows us to calculate traffic volumes, prices, revenues of operators and surpluses of consumers in four states of the market:

- the state of the market in 2003;
- the state of saturation, under zero prices;
- the social optimum state, where retail and wholeslale prices are equal to marginal technical costs;
- the state of the monopolistic equilibrium under regulation of call termination tariffs.

In each of these states we normalize the fixed part of the utility equal to the fixed part of the affine tariff: $u_i = f_i =$€10/month.

Market state of 2003

$$\mathbf{p} = \begin{bmatrix} 18 & 18 & 18 \\ 18 & 18 & 18 \\ 18 & 18 & 18 \end{bmatrix} \quad (\text{c€}) \ , \ \mathbf{Q} \ = \begin{bmatrix} 14\,153 & 4\,380 & 2\,052 \\ 4\,244 & 8\,726 & 1\,439 \\ 1\,809 & 1\,308 & 3\,058 \end{bmatrix} \quad (\text{Mmn}) \ .$$

The revenues of three operators are established respectively at €6 105 M, €4 274 M et €1 832 M, which corresponds to an ARPU (Average Revenue per User) per month of approximately €25, noticeably lower than the utility per client and per month equal to €41. Correspondingly, the consumption surplus is €16 per client and per month.

The interconnection balance (i.e. the revenue from the incoming interconnection minus the payment for outgoing interconnection) is negative for the two first operators (resp. €−221 M and €−109 M) while positive for the third one (€+329 M).

Saturation state

$$\widehat{\mathbf{p}} = \mathbf{0} \quad , \quad \widehat{\mathbf{Q}} = \begin{pmatrix} 21\,968 & 6\,237 & 2\,673 \\ 6\,237 & 13\,507 & 1\,871 \\ 2\,673 & 1\,871 & 4\,719 \end{pmatrix} \ (\text{Mmn}) \ .$$

At the saturation state, in other words if consumers can communicate for free, the outgoing traffic volumes of each operator will be 50% higher than their levels observed in 2003. The surplus of consumption per client and per month will be as high as €35 for each operator; hence, the relative gain of the surplus compared to the actual state of the market in 2003 (€16 per client and per month) will be higher than 100%.

Social optimum

$$\mathbf{p}^* = \begin{pmatrix} 2.9 & 2.9 & 2.9 \\ 2.9 & 2.9 & 2.9 \\ 2.9 & 2.9 & 2.9 \end{pmatrix} \ (\text{c€}) \ , \ \mathbf{Q}^* = \begin{pmatrix} 20\,726 & 5\,942 & 2\,574 \\ 5\,920 & 12\,747 & 1\,802 \\ 2\,536 & 1\,782 & 4\,456 \end{pmatrix} \ (\text{Mmn}) \ .$$

With marginal costs of the traffic being low, the retail prices are low as well. The traffic matrix at the social optimum is therefore close to the saturation matrix and the consumer surplus per client and per month, €31, 5, is close to the one obtained in the state of saturation (€35). The tariffs of call termination being equal to marginal costs, interconnection balances are small in absolute value, −€0.9 M, €0 M et +€0.9 M respectively.

Regulated monopolistic equilibrium

$$\overline{\mathbf{p}} = \begin{pmatrix} 27 & 39 & 47 \\ 37 & 27 & 47 \\ 37 & 38 & 27 \end{pmatrix} (\text{c€}) \quad , \quad \overline{\mathbf{Q}} = \begin{pmatrix} 10\,716 & 2\,016 & 682 \\ 1\,978 & 6\,631 & 481 \\ 836 & 599 & 2\,331 \end{pmatrix} (\text{Mmn}) \ .$$

The equilibrium prices, especially off-net prices, are high since they are driven up by a strong psychological valorization of a call minute, as well as by a high mark-up on call termination tariffs, especially for the third operator. As a result, equilibrium traffic volumes are lower than the levels observed in 2003 – approximately by third – and the traffics sent to the smallest operator are the lowest.

The real market of 2003 is then more “competitive” than it would be in a hypothetical equilibrium where the market shares of operators would be “protected” by a collusion and/or by the absence of market fluidity. In reality, the market shares are not protected but are “contestable” and each operator, being afraid to loose clients, sets the prices significantly lower than in the monopolistic equilibrium. In a monopolistic equilibrium, the consumer surplus per client and per month will be €6, or approximately 3 times lower than on the real market (€16).

Even though they are regulated, the call termination tariffs of 2003 stay significantly higher than the marginal costs, which generates in the equilibrium an interconnection balances significantly different from zero, −€32 M, −€10 M and +€42 M respectively. Under a strict cost orientation of wholesale tariffs, these equilibrium balances would be considerably reduced, to €−0.4 M, €0.0 M and +€0.4 M respectively.

The table below sums up the revenues, surpluses and interconnection balances in four states of the market described above.

	Utility (€/client/month)	Revenue (€/client/month)	Surplus (€/client/month)	Interconnection balance (€M)
2003	41	25	16	−221 | −109 | 329
Saturation	45	10	35	–
Optimum	45	13.5	31.5	−0.9 | 0 | +0.9
Equilibrium	33	27	6	−32 | −10 | 42

3.2 Three alternative price changes

Let us compare the effects of three alternative price changes from the initial market state corresponding to the French mobile market in 2003. We assume that each change is *neutral*, the retail revenues of operators and so the clients' spendings being maintained at a constant level, an increase of the fixed part f_i of the affine tariff compensating the decrease of minute prices $\mathbf{p}_i$.

By the duality between affine pricing and package pricing, this hypothesis of neutrality reflects the wide spread practice of operators, consisting in relaxing the volume constraint of package without changing its price.

- the first transition (T_1) consists in simultaneous on-net minute price cuts of 100%, as in the scenario of massive introduction of unlimited on-net calls by each of the three operators;
- the second transition (T_2) consists in simultaneous minute price cuts in all the traffic directions (all-net cuts) that generate the same consumer surplus variation as (T_1) (and so the same utility variation, since the bill does not change) ;
- the third transition (T_3) consists in, similarly to (T_2), simultaneous all-net price cuts that generate the same retail and wholesale profit variation as (T_1).

Transition (T_1)*: on-net calls free of charge.*

If an on-net minute is for free, consumers strongly increase their individual on-net traffics (approximately by 30 mn per month): on-net traffics of operators 1, 2 and 3 respectively increase by 8 496 Mmn, 5 286 Mmn and

1 900 Mmn per year, which entails the gains in consumer surplus of €642 M, €385 M and €128 M respectively (of €32.1 , €27.5 and €21.3 per consumer).

In this on-net transition, the flux of off-net traffic slightly decreases, by the effect of substitution of off-net by on-net. The interconnection balances vary consequently, respectively by +€59 M, +€11 M et −€70 M. Because of additional traffic sent and received through the network, the technical costs of operators increase: consequently, the profits of operators decreases by €168 M (€8.4/subscriber), €125 M (€9/subscriber) and €114 M (€19 /subscriber) respectively. The net gain in the social welfare, with respect to the initial situation, is €747 M, reflecting the gain of €1 155 M for consumers and the loss of €408 M for operators. Relative to the size of client bases, this loss is twice less severe for operators 1 and 2 than for operator 3.

Transition (T_2): "consumer-equivalent" all-net price cuts.

For the subscribers, this all-net transition is, by construction, strictly equivalent to the on-net transition (T_1) but the traffic fluxes increase not only in the on-net direction but also in the off-net directions.

The consumer-equivalence condition leads to the price cuts of −8 c, −7 c and −5 c per all-net minute (−44.6%, −36.3% and −26.5%). It results in the increases in the total outgoing traffic of 4 590 Mmn, 2 613 Mmn and 818 Mmn, approximately two times lower than on-net traffic increases produced by the transition (T_1). This means that the consumers are inclined to exchange "quantity" against "quality": a twice less traffic increase is enough to give them the same increase in utility when it is distributed in *all-net* and not concentrated in *on-net*

The biggest operator 1 looses in the interconnection balance (−€47 M) in favor of two small operators 2 and 3 (*resp.* +€5 M and +€42 M). This effect can be partially explained by the asymmetry of call termination tariffs in favor of the operator 3. The variations in total profits are −€176 M (−8.8 €/subscriber), −€71 M (−5.1 €/subscriber) and +€18 M (+3 €/subscriber) respectively. The transition (T_2) is, hence, slightly more unfavorable to the biggest operator 1 than the transition (T_1); on the other side, this transition is clearly more favorable to operators 2 and 3.

Compared to the initial situation, the transition (T_2) generates a gain of social welfare of €926 M, or €179 M more than the transition (T_1), exclusively to the profit of the operators. This gain of €926 M consists of utility gain of €1 155 M for the consumers, like in transition (T_1), and of loss equal to €229 M for the operators, against €407 M under (T_1); nevertheless, the profit distribution among operators differs.

Transition (T_3): all-net "operator-equivalent"cuts.

For each operator this *all-net* transition is, by construction, strictly equivalent to the *on-net* transition (T_1) in terms of losses of total profit.

On the other hand, for consumers, the increase in utility gains compared to transitions (T_1) and (T_2) is very significant: for the clients of operator 1 the utility gain equals €825 M in the transition (T_3) against €642 M in (T_1) and (T_2), or +28.5% ; for the clients of operator 2, it equals €566 M against €385 M, or +47% ; and, more importantly, for the clients of operator 3, it becomes €274 M compared to €128 M, or +114%. These utility gains come from an important increase in outgoing traffic, *resp.* +6 885 Mmn, +4 633 Mmn, and +2730 Mmn, allowed by cuts in *all-net* minute prices which are more significant than in "consumer-equivalent" transition (T_2), respectively −66.9%, −64.3%, and −88.4% against −44.6%, −36.3%, and −26.5%.

In order to understand the reason of such "performance" of the transition (T_3), it is useful to imagine the sliding of the transition (T_2) to the transition (T_3). In this sliding everything goes as though the small operator 3 used its positive "reserve" of profit, aquired in (T_2) thanks to the interconnection, in order to achieve in (T_3) two objectives for the good of society: on the one hand, cut its *all-net* minute prices more than its competitors 1 and 2; on the other hand, partially reduce the losses of those two operators (decreased, respectively, by €8 M and €54 M compared to the transition (T_2)) and give them reserves to reduce their *all-net* prices.

Compared to the initial situation, the transition (T_3) increases the social welfare by €1 258 M; the consumers gain €1 665 M and the operators lose €407.5 M. This transition is also Pareto-superior to (T_1), because the operators are indifferent between (T_1) and (T_3) and the consumers strongly prefer (T_3). In addition, compared to (T_2), the transition (T_3) generates a pure gain of collective suplus of €332 M, which benefits the consumers exclusively and adds to the transfer of €178 M coming from the operators.

The following table summarizes the characteristics of three alternative transitions.

	ΔProfit €M	ΔCons. surplus €M	(€/pers)	ΔSocial welfare €M
Transition (T_1)	−168	642	(32.1)	747
	−126	385	(27.5)	
	−114	128	(21.3)	
Transition (T_2)	−176	642	(32.1)	926
	−71	385	(27.5)	
	+18	128	(21.3)	
Transition (T_3)	−168	825	(41.2)	1258
	−126	566	(40.4)	
	−114	274	(45.7)	

We deduce three results from this quantitative study.

- First, for the same increase in their communication volumes, the consumers gain more utility if this increase is distributed between all traffic directions rather than concentrated on the *on-net* direction. Hence, a transition (T_4), that consists in simultaneous *all-net* tariff cuts that generate for each operator the same traffic volume variations as in the *on-net* transition (T_1), would give to the consumers €1 666 M against €1 155 M (+44%).

- Next, *all-net* cuts increase the social welfare more than "equivalent" *on-net* cuts. The difference in gains is much more important when those decreases benefit the consumers and leave the operators indifferent (+68%), compared to the situation where they benefit the operators and leave the consumers indifferent(+24%).

- Finally, the smallest operator is both the most "exposed", *i.e.* it suffers the biggest losses per subscriber under the *on-net* cuts, and the most "driving", *i.e.* it generates the best performances of operator-equivalent cuts compared to consumer-equivalent ones. In fact it, by "renouncing" the increase of consolidated profit that would be procured to him by its positif balance of interconnection under a consumer-equivalent cut, "finances" a strong increase of the utility of its own clients and improves the situation of its competitors, indirectly allowing them to improve the situation of their own clients.

3.3 A scenario of consecutive changes

The following numerical simulation is inspired by the French mobile market in 2004, when two dominating operators reduced *on-net* tariffs significantly by introducing the offers including "unlimited" *on-net* calls and thus installing an *on-net/off-net* price difference, while the third operator opted for an *all-net* tariff structure.

We take the transition (T_1) (*cf. supra*) when all operators start to propose free on-net minutes as a reference scenario. This transition translates into moderate profit losses, €−168 M, €−126 M and €−114 M for the operators 1, 2, and 3, and into gains of €642 M, €385 M and €128 M for their respective clients.

Now suppose that two dominating operators decide to persue this plan of 100% on-net cut, but the third one decides to keep the *all-net* price structure and so adopts a neutral *all-net* cut which transfers to its clients the same utility gain as would have transfered the 100% *on-net* cut. By construction, from the consumers' point of view these price changes are equivalent to the transition (T_1). On the other side, because of differential increase of incoming interconnection, the dominant operators loose a little less of profit than they would in the reference transition (T_1), respectively €−112 M against €−168 M (€+56 M) and €−85 M against €−126 M (€+41 M), while the third operator looses more, €−187 M against €−114 M (€−73 M).

Let us imagine now that two dominant operators, after having discovered the actions of the third one, decide in their turn to adopt the *all-net* tariffs which preserve the utility and bills of their clients. These actions leave the situation of all consumers the same as before. The interconnection balance of big operators 1 and 2 degrades while the small operator 3 enjoys its increase; the respective variations in consolidated profits are €−64 M, €+14 M, and €+205 M.

Finally, all three operators decide to adjust their respective *all-net* prices in order to align to the profits they would have got under the transition (T_1). By doing so, thanks to a strong increase in social welfare (€+332 M), they provide the utility gains of €+183 M, €+181 M, and €+146 M to their respective clients, or much more than pure reimboursement of €178 M of profit.

The three stages of this chain of price transitions are summarized in the following table where we have put the differences between each stage and the following one.

	ΔProfits (€M)	ΔCons surplus. (€M)	ΔCol surplus. (€M)
Ref Scenario.(T_1)	$-168 \mid -126 \mid -114$	$642 \mid 385 \mid 128$	747
Stage 1 / Réf.	$+56 \mid +41 \mid -73$	$0 \mid 0 \mid 0$	+24
Stage 2 / Etape1	$-64 \mid +14 \mid 205$	$0 \mid 0 \mid 0$	+155
Stage 3 / Stage 2	$+8 \mid -54 \mid -133$	$+183 \mid +181 \mid +146$	+331
Stage 3 / Ref	$0 \mid 0 \mid 0$	$+183 \mid +181 \mid +146$	+510

This simulation justifies the Pareto-superiority of *all-net* price cuts over *on-net* ones and it reveals that if at the beginning of 2000*s* the three operators on the French mobile market (and not only the third one) had adopted *all-net* cuts rather than *on-net* ones, then the consumers, especially the clients of the third operator, would have obtained the substantially greater gains of surplus: in our model, all subscribers would have globally accumulated €1 665 M against €1 155 M, or an increase of 44%; and for the clients of the third operator €374 M against €128 M, or nearly a triple. Finally, the increase of social welfare would have been €1 257 M, against only €747 M under on-net price cut, or 68% of increase.

4 Conclusion

In this article, we analyzed a national market of mobile telephony using a modeling architecture that is general enough to account for the real market characteristics without excessive simplification and is operable enough to allow for simulations. Our model incorporates demand saturation effect, substitution between traffic directions, call externality, as well as "club effects", meaning that a user tends to call more regularly her/his friends and family who are often subscribed to the same network. The model describes the impact of a price variation in a given traffic direction (on-net or off-net) on the traffic volume in different directions and on the gains of different market participants.

Using our model we first re-established and clarified different results from the existing theoretic literature. To do so we thoroughly characterized two remarkable states of the market: on the one hand, the social optimum, in which the price per minute is set at the marginal social cost, i.e. equal to the marginal transmission cost net of the call externality (benefit from receiving calls); on the other hand, the market equilibria, with and without a regulation of call termination rates in the wholesale market.

Then, we carried out a comprehensive study of the impact — on the operators' profit and on the consumers' surplus — of an important class of price changes consistent with the current practice of operators, namely "neutral" changes that leave unaffected operators' retail revenues and consequently users' bills. Among neutral price changes we examined in particular: on the one hand, the relaxation of the volume constraint in a packaged offer (all-net price cut); on the other hand, the introduction of unlimited on-net calls (on-net price cut). Our main conclusion is that, contrary to on-net price cuts, all-net price cuts simultaneously practiced by all operators do increase interconnection profits and thus constitute a provision allowing operators to reduce retail profits and to increase consumers' surplus.

Finally, we have numerically calibrated our model from the data of the French mobile market in 2003, then have used this calibration to get quantitative results on the impacts of price changes. According to our results, consumers show great "preference for the variability", in the sence that an *all-net* and *off-net* increase of traffic distributed in all directions gives them twice the increase in utility compared to the same increase concentrated only the *on-net* direction. *On-net* price cuts are found to have a significantly greater negative impact on smaller operators in terms of profit per subscriber. The *all-net* price cuts made by all operators simultaneously are preferable to simultaneous *on-net* cuts, since the former produce more gains in surplus of consumers for the same loss of profit by operators, or reduce the losses of operators for the same increase in consumers' surplus. The relative gain produced by simultaneous *all-net* cuts compared to simultaneous *on-net* cuts is significantly higher when the beneficiaries of this gain are the consumers rather than operators. Finally, we conclude that on the French market, where in 2004 two first entrants opted for important *on-net* cuts and the third entrant adopted the *all-net* price structure, it would have been preferable if all operators had decided to make *all-net* cuts. This strategy would have brought, in terms of differential gain of collective welfare, around 70%.

5 Appendix

5.1 Data and assumptions for calibration

The reference market is the French mobile market in 2003, the year preceding introduction of important *on-net* price reduction. Publicly available numerical data as well as ower main hypotheses are the following.

1. In 2003, the French mobile customer base counts about $M = 40$ million subscribers, distributed between $M_1 = 20$ millions for the incumbent France Télécom (Orange), $M_2 = 14$ milllions for SFR and $M_3 = 6$ millions for the last entrant, Bouygues Télécom.

2. According to ARCEP, the total mobile traffic volume is equal to 41 170 Mmn (millions of minutes). Supposing that the traffic sent per customer is the same for each operator, $q = Q/M = 1029$ mn, outgoing traffics are $Q_1 = 20\,585$ Mmn, $Q_2 = 14\,410$ Mmn and $Q_3 = 6\,176$ Mmn.

4. Public data on the revenues of operators and on their offers leads to the following estimations of the fixed component of the package price and of the all-net price per minute: $f_1 = f_2 = f_3 = 10$ €/client/mois and $p_1 = p_2 = p_3 = 18$ c€.

5. Statistics of ARCEP give the part s of on-net traffic in the total mobile to mobile traffic in 2003: $s = Q_+/Q = 0.63$.

6. The marginal costs of sending or receiving traffic and the call termination tariffs are also published by ARCEP: $c_1 = c_2 = c_3 = 1.43$ c€, $\tau^1 = 17.07$ c€, $\tau^2 = 17.07$ c€, $\tau^3 = 24.67$ c€.

7. Econometrical studies of the demand for telecommunications provide a large interval for the price elasticity, typically going from -1 to 0.[9] At the same time, in our linear model of demand the elasticities vary depending on the state of the market: they are close to zero if all the prices are very low and the volumes are very high, as in the neighborhood of the social optimum; they are on the contrary strong in absolute value if the prices are high and the volumes are low, as in the neighborhood of the monopolistic equilibrium. We will set the absolute value e of elasticity of the total outgoing traffic to all-net price in 2003 at an intermediary level -0.5, the same for each operator. This choice generates a big variability of elasticities depending on the state of the market, going from -0.04 in the social optimum and -1.4 in the equilibrium, that reflects a wide range of econometric estimations.

8. We will assume that a client of a given operator "perceives" the park of this operator and the one of other operators taking into account the distribution of the potential called parties among operators. Besides, called

[9] See Kri-del, Rappoport et Taylor (2002), Rappoport et Taylor (1997), Vodafone (2007), Grzybowski (2004), Rodini et alii (2002), Karacucka et alii (2002).

parties of an individual belong to two groups: on the one hand, a group of "privileged" called parties – "friends and family" – its proportion z is the same for all the consumers, and all of them are subscribed to the same operator as the consumer him/herself; on the other hand, a group of more occasional called parties, distributed among all the operators in proportion of their client bases. Let Z_i denote the numbre of potential called parties of the i-representative and $Z_{j/i}$ – the number of called parties subscribed to the operator j:

$$\forall j : \frac{M_{j/i}}{M} = \frac{Z_{j/i}}{Z_i} \quad , \quad \frac{Z_{i/i}}{Z_i} = z + (1-z)\frac{M_i}{M} \quad , \quad j \neq i : \frac{Z_{j/i}}{Z_i} = (1-z)\frac{M_j}{M} \ .$$

From it we deduce the matrix $M_{j/i}$ of perceived client bases as well as the deformation matrix $\omega_{j/i}$, introducing notation $M_{-i} = M - M_i$:

$$M_{i/i} = zM + (1-z)M_i = M_i + zM_{-i} \quad , \quad M_{j/i} = (1-z)M_j$$
$$\omega_{i/i} = 1 + z\frac{M_{-i}}{M_i} > 1 \quad , \quad \omega_{j/i} = 1 - z < 1 \ .$$

9. We assume here, for every operator, that the substitution between traffic directions is uniform. In the absence of specific information on the substitution structure this hypothesis appears as the most "neutral". We obtain:

$$k \neq j \neq i : \xi_i^{i,j} = \xi_i^{i,k} = \xi_i^{j,k} = \xi_i \ ,$$

from which, by the inversion of matrix $\mathbf{A}_i = \mathbf{X}_i{}^{-1}/v_i$:

$$\alpha_i^j = \frac{1}{v_i}\frac{1+\xi_i}{(1-\xi_i)(1+2\xi_i)} \quad , \quad \beta_i^{i,j} = \beta_i^{i,k} = \beta_i^{j,k} = \frac{1}{v_i}\frac{\xi_i}{(1-\xi_i)(1+2\xi_i)}$$

We have fixed the uniform substitution coefficient equal to $\xi_i^{j,k} = \xi = 0.1$, which corresponds to a reasonable order of magnitude for the ratio between proper price elasticity and cross-price elasticity.

10. We fix the median value $a = 0.5$ for the coefficient of appetence for the number of call receivers. It corresponds to the psychological value of the "first" communication minute independant of the size of the destination network and so of the traffic direction.

11. We neglect the externality of receiving calls ($\rho_i = 0$) since to our knowledge no econometrical study has found it to be significantly different from zero.[10]

[10]See Sanbach et Van Hooft (2008) and Frontier Economics (2010).

5.2 Calibration procedure

Let us introduce the following notation:

$$\widetilde{\alpha}_i^j = \frac{1}{v_i}\frac{(1+\xi_i) - \left[(M_{k/i}/M_{j/i})^a + (M_{l/i}/M_{j/i})^a\right]\xi_i}{(1-\xi_i)(1+2\xi_i)} \ .$$

We use the demand functions (7) and the constraint of compatibility between the observed total outgoing traffic volumes Q_i and of the elasticity $-e_i$ of this traffic under *all-net* prices to obtain the expressions of parameters v_i and σ_i as a function of observed M, M_i, Q_i, p_i as well as of two parameters e_i and ξ_i:

$$v_i = \frac{p_i}{M}\frac{1+e_i}{e_i}\frac{(1+\xi_i)\sum_{k=1}^{3} M_{k/i}^{2a} - 2\xi_i \sum_{l<k} M_{k/i}^a M_{l/i}^a}{(1-\xi_i)(1+2\xi_i)} \quad , \quad \sigma_i = \frac{(1+e_i)Q_i}{MM_i} \ .$$

Substituting these different expressions to demand functions, we obtain coefficients of price sensitivity as well as the matrix of traffic in the scenario of uniform substitution:

$$\alpha_i^1 = \alpha_i^2 = \alpha_i^3 = \frac{M}{p_i}\frac{e_i}{1+e_i}\frac{1+\xi_i}{(1+\xi_i)\sum_{k=1}^{3} M_{j/i}^{2a} - 2\xi_i \sum_{l<k} M_{k/i}^a M_{l/i}^a}$$

$$\beta_i^{1,2} = \beta_i^{1,3} = \beta_i^{2,3} = \frac{M}{p_i}\frac{e_i}{1+e_i}\frac{\xi_i}{(1+\xi_i)\sum_{k=1}^{3} M_{j/i}^{2a} - 2\xi_i \sum_{l<k} M_{k/i}^a M_{l/i}^a}$$

$$\widetilde{\alpha}_i^j = \frac{M}{p_i}\frac{e_i}{1+e_i}\frac{1+\left[1-(M_{k/i}/M_{j/i})^a - (M_{l/i}/M_{j/i})^a\right]\xi_i}{(1+\xi_i)\sum_{k=1}^{3} M_{k/i}^{2a} - 2\xi_i \sum_{l<k} M_{k/i}^a M_{l/i}^a}$$

$$Q_i^j = Q_i\frac{M_{j/i}}{M}\left[1+e_i\left(1-\frac{M}{M_{j/i}^{1-a}}\frac{M_{j/i}^a + [M_{j/i}^a - M_{k/i}^a - M_{l/i}^a]\xi_i}{(1+\xi_i)\sum_{k=1}^{3} M_{k/i}^{2a} - 2\xi_i \sum_{l<k} M_{k/i}^a M_{l/i}^a}\right)\right] \ .$$

Besides, under the hypothesis of the same price elasticity $e_i = e$ and of the same substitution coefficient $\xi_i = \xi$ for each operator, the ratio s of the total on-net traffic to the total outgoing traffic equals

$$s = \frac{1}{Q}\sum_{i=1}^{n} Q_{i/i} \ .$$

This relation, in which we substitute the expression of $Q_{i/i}$ obtained above, replacing $M_{i/i}$ by $M_i + zM_i$ and $M_{j/i}$ by $(1-z)M_j$, allows to calculate the ratio "friends and family" z from the observed ratio s and from observed parameters e and ξ.

From the data given in the preceeding section, we deduce the numerical values of all the model parameters.

"Friends and family"rate: $z = 42\%$.

Psychological values of a minute: $v_1 = 47.4$c€, $v_2 = 46.8$c€, $v_3 = 45.5$c€.

Individual communication potentials: $\sigma_1 = \sigma_2 = \sigma_3 = 38.6$ mn.

Deformation coefficients and perceived client bases.

$$\omega_{./.} = \left[\omega_{j/i}\right]_{i,j} = \begin{pmatrix} 1.42 & 0.58 & 0.58 \\ 0.58 & 1.79 & 0.58 \\ 0.58 & 0.58 & 3.40 \end{pmatrix}$$

$$\begin{aligned} \mathbf{M}_{./1} &= \text{Diag}[28.46\,;\,8.08\,;\,3.46]\ \text{(Millions)} \\ \mathbf{M}_{./2} &= \text{Diag}[11.54\,;\,25.00\,;\,3.46]\ \text{(Millions)} \\ \mathbf{M}_{./i} &= \text{Diag}[11.54\,;\,8.08\,,\,20.38]\ \text{(Millions)}\ . \end{aligned}$$

Proper and cross-coefficients of price sensitivity.

$$\mathbf{A}_1 = \begin{bmatrix} 2.149 & -0.195 & -0.195 \\ -0.195 & 2.149 & -0.195 \\ -0.195 & -0.195 & 2.149 \end{bmatrix} , \quad \mathbf{A}_2 = \begin{bmatrix} 2.174 & -0.198 & -0.198 \\ -0.198 & 2.174 & -0.198 \\ -0.198 & -0.198 & 2.174 \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} 2.24 & -0.20 & -0.20 \\ -0.20 & 2.24 & -0.20 \\ -0.20 & -0.20 & 2.24 \end{bmatrix} \quad (€^{-1})\ .$$

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